Name______________________ Date______________ Period______

Worksheet ~~3.3~~ 42—Increasing, Decreasing, and 1st Derivative Test

Show all work. No calculator unless otherwise stated.

Multiple Choice

_____ 1. Determine the increasing and decreasing open intervals of the function $f(x)=(x-3)^{4/5}(x+1)^{1/5}$ over its domain. Tip: factor out least powers from the derivative to put it into full-fledged-factored-form!

$f'(x) = \frac{1}{5}(x-3)^{4/5}(x+1)^{-4/5} + \frac{4}{5}(x-3)^{-1/5}(x+1)^{1/5}$

$\frac{1}{5}(x+1)^{-4/5}(x-3)^{-1/5}\left[(x-3)+4(x+1)\right]$

$\frac{1}{5}(x+1)^{-4/5}(x-3)^{-1/5}(5x+1)$

(A) Inc: $\left(-1,-\frac{1}{5}\right)$, Dec: $\left(-\frac{1}{5},\infty\right)$

(B) Inc: $\left(-1,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(C) Inc: $(-\infty,-1)\cup(3,\infty)$, Dec: $(-1,3)$

(D) Inc: $\left(-\infty,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(E) Inc: $\left(-\frac{1}{5},3\right)\cup(3,\infty)$, Dec: $\left(-1,\frac{1}{5}\right)\cup(3,\infty)$

$(x+1)^{-4/5}$: + | + | + | +

$(x-3)^{-1/5}$: − | − | − | +

$5x+1$: − | − | + | +

(+) −1 (+) $-\frac{1}{5}$ (−) 3 (+)

Note: $(x+1)^{-4/5}$ always ≥ 0

_____ 2. Let f be the function defined by $f(x)=x-\cos 2x,\ -\pi\leq x\leq\pi$. Determine all open interval(s) on which f is decreasing. $f'(x)<0$

$f'(x) = 1 + 2\sin 2x$

Note:

$-\pi \leq x \leq \pi$

$-2\pi \leq 2x \leq 2\pi$

$1 + 2\sin 2x = 0$

$\sin 2x = -\frac{1}{2}$

$2x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = -\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

(A) $\left(-\frac{5\pi}{12},-\frac{\pi}{12}\right)$, $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$

(B) $\left(-\frac{5\pi}{12},-\frac{\pi}{6}\right)$, $\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$

(C) $\left(-\frac{5\pi}{12},-\frac{\pi}{8}\right)$, $\left(\frac{3\pi}{8},\frac{11\pi}{12}\right)$

(D) $\left(-\frac{\pi}{6},-\frac{\pi}{12}\right)$, $\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$

(E) $\left(-\pi,-\frac{5\pi}{12}\right)$, $\left(\frac{7\pi}{12},\pi\right)$

$f'(x)$: + | − | + | − | +

$-\pi$, $-\frac{5\pi}{12}$, $-\frac{\pi}{12}$, $\frac{7\pi}{12}$, $\frac{11\pi}{12}$, π

3. Let $f(x) = x\left(4 + x^2 - \frac{x^4}{5}\right)$.

$f'(x) = x\left(2x - \frac{4}{5}x^3\right) + \left(4 + x^2 - \frac{1}{5}x^4\right)$

$= 2x^2 - \frac{4}{5}x^4 + 4 + x^2 - \frac{1}{5}x^4$

$= -x^4 + 3x^2 + 4$

$= (4 - x^2)(1 + x^2)$

always > 0

_____ (i) Which of the following is $f'(x)$?

(A) $f'(x) = (1+x^2)(5-x^2)$

(B) $f'(x) = (1+x^2)(4-x^2)$

(C) $f'(x) = (1-x^2)(5+x^2)$

(D) $f'(x) = (1-x^2)(4+x^2)$

(E) $f'(x) = (1-x^2)(4-x^2)$

_____ (ii) Find the open interval(s) on which f is increasing.

(A) $(-\infty,-2)\cup(2,\infty)$

(B) $(-\infty,-\sqrt{5})\cup(\sqrt{5},\infty)$

(C) $(-2,2)$

(D) $(-\infty,-1)\cup(1,\infty)$

(E) $(-1,1)$

$2-x$	+		+		−
$2+x$	−		+		+
$1+x^2$	+		+		+
	⊖	-2	⊕	2	⊖

$2(x^2+2x-8)$

_____ 4. The derivative of a function f is given for all x by $f'(x) = (2x^2+4x-16)(1+g^2(x))$ where g is some unspecified function. At which value(s) of x will f have a local maximum?

Note: $g^2(x) = (g(x))^2$

(A) $x = -4$

(B) $x = 4$

(C) $x = -2$

(D) $x = 2$

(E) $x = -4,\ 2$

$x+4$	−		−		+
$x-2$	−		+		+
$1+g^2(x)$	+		+		+
	⊕	-4	⊖	2	⊕
	↗		↘		↗

_____ 5. Which of the following statements about the absolute maximum and absolute minimum values of $f(x)=\dfrac{x^3-4x^2-6x-1}{x+1}$ on the interval $[0,\infty)$ are correct? (Hint: Think of what type of discontinuity does $f(x)$ have??? $\frac{0}{0}$ or $\frac{\neq 0}{0}$)

$-1)\ \ 1 \ \ -4 \ \ -6 \ \ -1$

$\quad\ \ -1 \ \ 5 \ \ 1$

$\quad 1 \ \ -5 \ \ -1 \ | \ 0$

(A) Max $=13$, No Min

(B) No Max, Min $=-\frac{29}{4}$

(C) Max $=13$, Min $=-\frac{29}{4}$

(D) Max $=5$, No Min

(E) No Max, Min $=-1$

$f(x)\ \ x^2-5x-1=0$

$f'(x)=2x-5$

$2x-5 \quad - \ | \ +$

$\searrow \ \frac{5}{2} \ \nearrow$

No max

min @ $f\left(\frac{5}{2}\right)=\dfrac{\frac{125}{8}-25-15-1}{\frac{5}{2}+1}$

$=\dfrac{\frac{125}{8}-\frac{328}{8}}{\frac{7}{2}}=\dfrac{-203}{8}\cdot\dfrac{2}{7}=-\dfrac{29}{4}$

_____ 6. (**Calculator Permitted**) The first derivative of the function f is defined by $f'(x)=\cos\left(x^3-x\right)$ for $0\le x\le 2$. On what intervals is f increasing?

(A) $0\le x\le 1.445$ only

(B) $1.445\le x\le 1.875$

(C) $1.691\le x\le 2$

(D) $0\le x\le 1$ and $1.691\le x\le 2$

(E) $0\le x\le 1.445$ and $1.875\le x\le 2$

$f'(x)>0$

Look @ graph of $f'(x)$

Short Answer

7. For each of the following, find the critical values (on the indicated intervals, if indicated.) Remember, a critical value MUST be in the domain of the function, though it may not be in the domain of the derivative.

(a) $f(x)=x^2(3-x)$

$= 3x^2 - x^3$

$f'(x) = 6x - 3x^2$

$= 3x(2-x)$

$x=0,\ x=2$

(b) $f(x)=\dfrac{\sin x}{1+\cos^2 x}$, $[0,2\pi]$

on back

(c) $f(x)=\dfrac{x^2}{x^2-9}$

on back

8. Determine the local extrema of each of the following functions (on the indicated interval, if indicated). Be sure to say which type each is. **Justify** (this means write a sentence.)

(a) $f(x)=\cos^2(2x)$, $[0,2\pi]$

$f'(x) = -2\cos(2x)\cdot\sin(2x)\cdot 2$

$= -4\cos(2x)\sin(2x)$

$\cos 2x = 0$ $\quad$ $\sin 2x = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $\quad$ $2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\quad$ $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(b) $f(x)=x+\dfrac{1}{x}$

$f'(x) = 1 - \frac{1}{x^2}$

$x = \pm 1$

(c) $f(x)=\sin^2 x+\sin x$, $[0,2\pi]$

$f'(x) = 2\sin x\cdot\cos x + \cos x$

$0 = \cos x(2\sin x + 1)$

$\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

(d) $f(x)=\dfrac{x^2-3x-4}{x-2}$

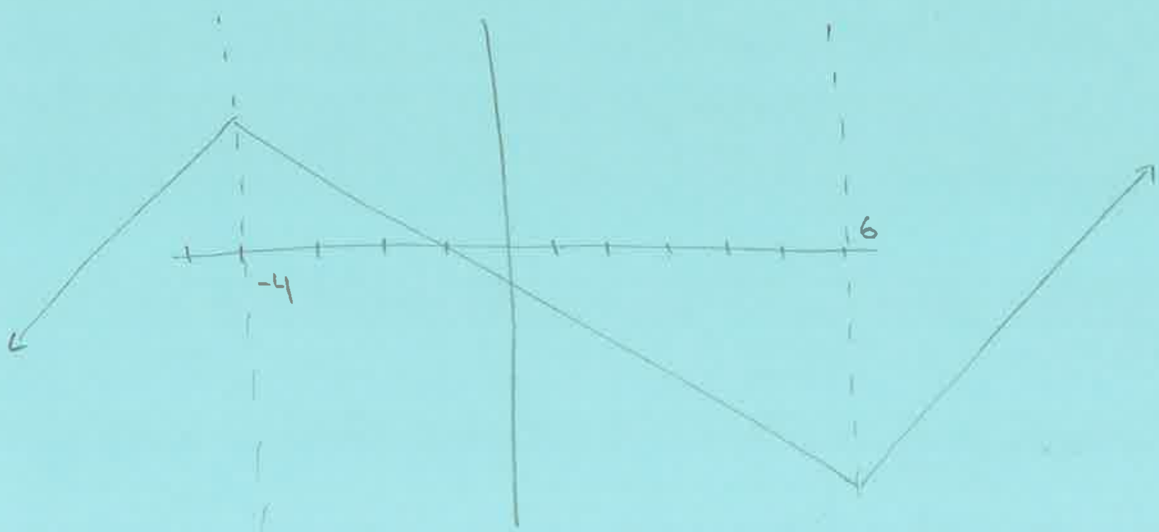
on back

9. Assume that f is differentiable for all x. The signs of f' are as follows.

$$f'(x) > 0 \text{ on } (-\infty, -4) \cup (6, \infty) \text{ and } f'(x) < 0 \text{ on } (-4, 6)$$

Let $g(x)$ be a transformation of $f(x)$. Supply the appropriate inequality ($>, <, \geq, \leq$) for the indicated value of c in the given blank.

Function	**Sign of $g'(c)$**
(a) $g(x) = f(x) + 5$ vert shift	$g'(0)$ $<$ 0
(b) $g(x) = 3f(x) - 3$ vert shift	$g'(-5)$ $>$ 0
(c) $g(x) = -f(x)$ flip	$g'(-6)$ $<$ 0
(d) $g(x) = -f(x)$ flip	$g'(0)$ $>$ 0
(e) $g(x) = f(x-10)$ slide	$g'(0)$ $>$ 0
(h) $g(x) = f(x-10)$ slide	$g'(8)$ $<$ 0
(i) $g(x) = f(-x)$	$g'(8)$ $>$ 0
(j) $g(x) = f(-x)$	$g'(-8)$ $>$ 0



7 b) $f(x) = \dfrac{\sin x}{1+\cos^2 x}$

$$f'(x) = \frac{\cos x\,(1+\cos^2 x) + (2\cos x \cdot \sin x)(\sin x)}{(1+\cos^2 x)^2}$$

$$= \frac{\cos x\left[1+\cos^2 x + 2\sin^2 x\right]}{b^2}$$

$$= \frac{\cos x\left[1+(1-\sin^2 x)+2\sin^2 x\right]}{b^2}$$

$$= \frac{\cos x\,(2+\sin^2 x)}{(1+\cos^2 x)^2}$$

$\cos x = 0$ $\quad 2+\sin^2 x = 0$ $\quad (1+\cos^2 x)^2 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ never $\quad$ never

c)

$f(x) = \dfrac{x^2}{x^2-9}$

$$f'(x) = \frac{2x(x^2-9) - x^2 \cdot 2x}{(x^2-9)^2}$$

$$= \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2}$$

$$= \frac{-18x}{(x^2-9)^2}$$

$x = 0$ $\quad$ BTW, $x = \pm 3$ not in domain of f.

8 d) $f(x) = \dfrac{x^2-3x-4}{x-2}$

$$f'(x) = \frac{(x-2)(2x-3) - (x^2-3x-4)}{(x-2)^2}$$

$$= \frac{2x^2 - 7x + 6 - x^2 + 3x + 4}{(x-2)^2}$$

$$= \frac{x^2-4x+10}{(x-2)^2}$$

$f'(x) = 0$ when $x^2 - 4x + 10 = 0$

$x = \dfrac{4 \pm \sqrt{16-40}}{2}$ $\quad$ No critical values!

In fact, $f'(x) > 0 \;\; \forall x \neq 2$

$f(x)$ is monotonic over its domain.